A Model of Keynes' Integration of Monetary and Value Theory

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Abstract

A model of Keynes' integration of monetary and value theory that incorporates the price of non-debt assets as well as the prices of consumption and investment goods is specified below. The dynamics by which short-run and long-run equilibrium are obtained within this model are examined, and it is demonstrated that the Marshallian roots of Keynes' general theory make a causal analysis of dynamic behavior possible within the analytic framework developed by Keynes throughout *The General Theory*.

A Model of Keynes' Integration of Monetary and Value Theory

Keynes summarized the independent and dependent variables in the analytical framework he had developed throughout *The General Theory* at the beginning of Chapter 18. He then outlined the aggregate model embodied in this framework as follows:

> Thus we can sometimes regard our ultimate independent variables as consisting of (1) the three fundamental psychological factors, namely, the psychological propensity to consume, the psychological attitude to liquidity and the psychological expectation of future yield from capital-assets, (2) the wage-unit as determined by the bargains reached between employers and employed, and (3) the quantity of money as determined by the action of the central bank; so that, if we take as given the factors specified above, these variables determine the national income (or dividend) and the quantity of employment. *But these again would be capable of being subjected to further analysis, and are not, so to speak, our ultimate atomic independent elements* [*emphasis* added]. (p. 246-7) [1]

In order to express the behavioral equations of the model described by Keynes in this paragraph in the terminology used by Keynes throughout *The General Theory* it is necessary to define the model's variables in terms of Keynes' chosen units of measurement. It is also necessary to incorporate the crucial role played by expectations in Keynes' general theory in determining *causality* as the system changes *through time*. (Blackford 2020a, pp. 20-77; 2019a; 2019b; 2019d)

I. Behavioral Equations

Keynes made use of but two units of measurement, money and the labor-

[1] See also Keynes (1936, p. 247).

unit, where Keynes defined the labor-unit as "an hour's employment of ordinary labour" (1936, p. 41). In specifying the behavioral equation of Keynes' model in terms of money and labor units we begin by a) dividing the money values of the rates at which consumption ($\boldsymbol{P_t^c C_t}$) and investment ($\boldsymbol{P_t^i I_t}$) goods are produced by what Keynes defined as the wage-unit ($\boldsymbol{W_t}$)—that is, "the money-wage of a labour-unit" (1936, p. 41)—in order to express the *rates* at which consumption ($\boldsymbol{C_t}$) and investment ($\boldsymbol{I_t}$) goods are produced in wage-units and b) summing to obtain the aggregate level of output/income measured in wage-units ($\boldsymbol{Y_t^w}$):[2]

$$(\mathbf{1}) \quad \boldsymbol{Y_t^w = P_t^c C_t / W_t + P_t^i I_t / W_t} \\ \boldsymbol{= C_t^w + I_t^w},$$

where $\boldsymbol{C_t^w}$ and $\boldsymbol{I_t^w}$ are the aggregate rates of consumption $\boldsymbol{C_t}$ and investment $\boldsymbol{I_t}$ goods production expressed in terms of 'wage-units'—that is, hours of ordinary labor per unit of time—and $\boldsymbol{P_t^c}$ and $\boldsymbol{P_t^i}$ are the (weighted average) prices of $\boldsymbol{C_t}$ and $\boldsymbol{I_t}$, respectively.[3]

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[2] This specification does not presume homogeneous consumption or investment goods. Any number of consumption and investment goods can be aggregated in this way simply by dividing the sum of the money values of the individual rates of consumption and investment goods production by $\boldsymbol{W_t}$ to obtain the hours of ordinary labor that correspond to the money value of the aggregate..

[3] It is rather misleading to refer to Keynes' wage-unit measure as "constant-wage-unit dollars" as Hansen did in 1953 (p. 44). When the value of a flow variable such as consumption, investment, income, or labor which is measured in money-units/time-unit (e.g., dollars/year) is divided by Keynes' wage-unit which is measured in money-units/labor-unit (e.g., dollars/hour-of-ordinary-labor) the money-units cancel and we are left with labor-units/time-unit or hours of ordinary labor per unit of time (e.g., (dollars/year)/(dollars/hour-of-ordinary-labor) = hour(s)-of-ordinary-labor/year). Similarly, when a stock variable such as debt, non-debt assets, or money

Similarly, the "psychological attitude to liquidity" is embodied in Keynes' liquidity-preference/money-demand function and is assumed to be an inverse function of the rate of interest ($\boldsymbol{R_t}$) and a direct function of income measured in wage-units $\boldsymbol{Y_t^w}$:

(**2**) $\boldsymbol{M_t^{wd} = m^d(Y_t^w, R_t)}$, $\boldsymbol{m_1^d > 0}$, $\boldsymbol{m_2^d < 0}$,

where $\boldsymbol{M_t^{wd}}$ is the nominal value of the stock of money demanded $\boldsymbol{M_t^d}$ divided by the wage-unit $\boldsymbol{M_t^d/W_t}$ which expresses the value of the stock of money demanded $\boldsymbol{M_t^{wd}}$ in terms of hours of ordinary labor.[4]

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which is measured in money-units (e.g., dollars) is divided by Keynes' wage-unit (e.g., dollars/hour-of-ordinary-labor) the money-units again cancel and we are left with labor-units or hours of ordinary labor (e.g.., dollars/(dollars/hour-of-ordinary-labor) = hour(s)-of-ordinary-labor). As a result, money-units (e.g., dollars), constant or otherwise, cancel and are not part of the unit of measurement when a quantity is measured in wage-units. It must be noted, however, that since Keynes' measure presumes that "different grades and kinds of labour … enjoy a more or less fixed relative remuneration" it is not clear that Keynes' measure is superior to the constant-dollar measure of neoclassical economics in light of the changes in relative remunerations for various kinds of labor that have occurred over the past fifty-odd years. See Piketty (Chap. 9) and Blackford (2018, pp. 1-9, 279).

[4] Keynes argued in *The General Theory* (p. 304) that the demand for money is a function of effective demand $\boldsymbol{Y_t^{we}}$ (**22**), and in his 1938 attempt to clarify the nature of the demand for money and its relationship to 'finance' Keynes argued that the demand for money "is a function of income and of business habits" (1938, p. 321-2). Since effective demand is only related to producers' demands for money and is not related to the demands of purchasers, I believe that the best way to incorporate these two aspects of Keynes understanding of the demand for money is to assume that the demand for money is a direct function of realized output/income $\boldsymbol{Y_t^w}$ (**1**) which is related to the demands of both produces and purchasers and to assume that changes in

Keynes assumed the stock of money to be exogenously "determined by the action of the central bank" in his summary above, but in 1937 he observed that "an illuminating way of expressing the liquidity-theory" is in terms of the willingness for the public "to become more or less liquid and ... the banking system ... to become more or less unliqulid." (p. 666) Hence, it is assumed that the quantity of money supplied by the financial system, measured in wage-units ($\boldsymbol{M_t^{ws}}$), is directly related to the rate of interest $\boldsymbol{R_t}$:

$$(\mathbf{3}) \quad \boldsymbol{M_t^{ws} = m^s(R_t)}, \qquad \boldsymbol{m^{s'} > 0},$$

where $\boldsymbol{M_t^{ws}}$ is the nominal value of the stock of money in existence ($\boldsymbol{M_t}$) divided by the money wage (as defined by the wage-unit) $\boldsymbol{W_t}$ which yields the stock of money supplied expressed in terms of hours of ordinary labor.

It is assumed that the existing stock of non-debt assets ($\boldsymbol{A_t}$) is exogenously determined. This stock, measured in wage-units ($\boldsymbol{A_t^{ws}}$), is given by:

$$(\mathbf{4}) \quad \begin{aligned} \boldsymbol{A_t^{ws}} &= \boldsymbol{P_t^a A_t / W_t} \\ &= \boldsymbol{A_t^w}, \end{aligned}$$

where $\boldsymbol{P_t^a}$ is the (weighted average) price of non-debt assets. The demand for non-debt assets measured in wage-units ($\boldsymbol{A_t^{wd}}$) is assumed to be inversely related to the price of non-debt assets $\boldsymbol{P_t^a}$, the rate of interest $\boldsymbol{R_t}$, and output/income:

$$(\mathbf{5}) \quad \boldsymbol{A_t^{wd} = a^d(P_t^a, R_t, Y_t^w)}, \qquad \boldsymbol{a_1^d,\ a_2^d < 0,\quad a_3^d > 0}.$$

It is also instructive, for expository purposes, to specify the *non-debt asset equilibrium function* in this model even though Keynes did not utilize this relationship. This function can be obtained by setting the supply of non-debt

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effective demand $\boldsymbol{Y_t^{we}}$ (**22**) have the effect of shifting the demand for money function $\boldsymbol{m^d(Y_t^w, R_t)}$ (**2**) by way of changes in the demand for 'finance'. See Bibow, Blackford (2019a; 2019c; 2019d), and Keynes (1937).

assets (**4**) equal to the demand for non-debt assets (**5**),

(**6**) $A_t^w = a^d(P_t^a, R_t, Y_t^w)$,

and solving for the equilibrium price of non-debt assets P_t^a as a function of the rate of interest R_t and stock of non-debt assets A_t^w:

(**7**) $P_t^a = a(R_t, A_t^w, Y_t^w), \quad a_1, a_2 < 0, \; a_3 > 0.$

The "psychological propensity to consume" is embodied in Keynes' consumption function. If it is assumed that the supply price of consumption goods (P_t^{cs}) is a direct function of the rate of consumption goods production C_t^w the supply price of consumption goods P_t^{cs} can be written as: [5]

(**8**) $P_t^{cs} = c^{sp}(C_t^w), \qquad c^{sp'} > 0.$

If it is also assumed that the demand price of consumption goods (P_t^{cd}) is an inverse function of the rate of consumption goods production C_t^w and a direct function of the level of income Y_t^w the demand price of consumption goods P_t^{cd} can be written as:

(**9**) $P_t^{cd} = c^{dp}(C_t^w, Y_t^w), \; c_1^{dp} < 0, \; c_2^{dp} > 0.$

Keynes' consumption function can then be obtained by equating the supply price of consumption goods P_t^{cs} (**8**) and the demand price of consumption goods P_t^{cd} (**9**) to obtain:

(**10**) $c^{sp}(C_t^w) = c^{dp}(C_t^w, Y_t^w) = P_t^c,$

and solving this equation for the demand for consumption goods C_t^{wd} as a function of income Y_t^w:

(**11**) $C_t^{wd} = c(Y_t^w), \quad 0 < c' < 1,$

[5] It is assumed that the supply-price functions c^{sp} (**8**) and i^{sp} (**13**) below are derived from their corresponding employment functions and, hence, are independent of industry output. See Keynes (1936, pp. 280-91).

where $\boldsymbol{c}$ denotes Keynes' aggregate consumption function, $\boldsymbol{C_t^{wd}}$ is the rate of consumption goods demanded at each level of income $\boldsymbol{Y_t^w}$ given the (weighted average) price of consumption goods $\boldsymbol{P_t^c}$ as determined by the supplies and demands in the markets for consumption goods (**8**) and (**9**), respectively, and it is assumed that the Marginal Propensity to Consume (MPC) $\boldsymbol{c'}$ lies between zero and one. Thus, Keynes' aggregate savings function ($\boldsymbol{s}$) is given by:

$$(\mathbf{12}) \quad \boldsymbol{S_t^w} = \boldsymbol{Y_t^w} - \boldsymbol{c}(\boldsymbol{Y_t^w}) = \boldsymbol{s}(\boldsymbol{Y_t^w}), \quad 0 < \boldsymbol{s'} < 1.$$

The "psychological expectation of future yield from capital assets" is embodied in Keynes' Marginal Efficiency of Capital (MEC) schedule which Keynes defined as: "The relation between the prospective yield of a capital-asset and its supply price or replacement cost." (1936, pp. 135-6) This schedule can be derived in a manner parallel to the derivation of Keynes' consumption function.

If it is assumed that the supply price of investment goods ($\boldsymbol{P_t^{is}}$) is a direct function of the rate of investment goods production measured in wage-units ($\boldsymbol{I_t^w}$) the supply price of investment goods $\boldsymbol{P_t^{is}}$ can be written as:

$$(\mathbf{13}) \quad \boldsymbol{P_t^{is}} = \boldsymbol{i^{sp}}(\boldsymbol{I_t^w}), \qquad \boldsymbol{i^{sp'}} > 0.$$

If it is also assumed that the demand price of investment goods ($\boldsymbol{P_t^{id}}$) is an inverse function of the rate of investment goods production $\boldsymbol{I_t^w}$ and the rate of interest $\boldsymbol{R_t}$ and a direct function of the price of non-debt assets $\boldsymbol{P_t^a}$ (Keynes 1936, p. 151) and the demand for consumption goods $\boldsymbol{c}(\boldsymbol{Y_t^w})$ (Keynes 1936, pp. 46, 210-12) the demand price of investment goods $\boldsymbol{P_t^{id}}$ can be written as:

$$(\mathbf{14}) \quad \boldsymbol{P_t^{id}} = \boldsymbol{i^{dp}}\left(\boldsymbol{I_t^w}, \boldsymbol{R_t}, \boldsymbol{P_t^a}, \boldsymbol{c}(\boldsymbol{Y_t^w})\right) = \boldsymbol{i^{dp}}(\boldsymbol{I_t^w}, \boldsymbol{R_t}, \boldsymbol{P_t^a}, \boldsymbol{Y_t^w}), \quad \boldsymbol{i_1^{dp}}, \boldsymbol{i_2^{dp}} < 0, \ \boldsymbol{i_3^{dp}} \ \boldsymbol{i_4^{dp}} > 0.$$

Keynes' MEC schedule can then be obtained by equating the supply price of investment goods $\boldsymbol{P_t^{is}}$ (**13**) and demand price of investment goods $\boldsymbol{P_t^{id}}$ (**14**) to

obtain:

$$(\mathbf{15}) \quad \boldsymbol{i^{sp}(I_t^w) = i^{dp}(I_t^w, R_t, P_t^a, Y_t^w) = P_t^i,}$$

and solving for Keynes' MEC schedule:

$$(\mathbf{16}) \quad \boldsymbol{I_t^{wd} = i(R_t, P_t^a, Y_t^w), \qquad i_1 < 0,\ i_2, i_3 > 0,}$$

where $\boldsymbol{i}$ denotes Keynes' MEC schedule, and $\boldsymbol{I_t^{wd}}$ is the rate at which investment goods are demanded at each rate of interest $\boldsymbol{R_t}$ given the exogenously determined stock of non-debt assets $\boldsymbol{A_t^w}$, the level of output/income $\boldsymbol{Y_t^w}$, and the (weighted average) price of investment goods $\boldsymbol{P_t^i}$ and non-debt assets $\boldsymbol{P_t^a}$ as determined by the supplies and demands in the markets for investment goods, and non-debt assets. [6]

Equations (**11**) and (**16**) imply that Keynes' aggregate demand function can be written as:

$$(\mathbf{17}) \quad \boldsymbol{Y_t^{wd} = C_t^{wd} + I_t^{wd}} \\ \boldsymbol{= c(Y_t^w) + i(R_t, P_t^a, Y_t^w),}$$

where $\boldsymbol{Y_t^{wd}}$ is the aggregate demand for output measured in wage-units.[7]

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[6] Cf., Keynes:

> There will be an inducement to push the rate of new investment to the point which forces the supply-price of each type of capital-asset to a figure which, taken in conjunction with its prospective yield, brings the marginal efficiency of capital in general to approximate equality with the rate of interest. That is to say, the physical conditions of supply in the capital-goods industries, the state of confidence concerning the prospective yield, the psychological attitude to liquidity and the quantity of money (preferably calculated in terms of wage-units) determine, between them, the rate of new investment. (1936, p. 248)

[7] It should be noted that any number of individual consumption or investment goods can be aggregated in this way and that to simplify the exposition it is assumed that

To complete behavioral equations in Keynes' aggregate model it is necessary to explain the relationship between the level of employment and effective demand where Keynes defined *effective demand* as the point at which the "entrepreneurs' expectation of profits will be maximized" (1936, p. 25). Keynes explained this relationship in Chapter 20 in terms of his employment function:

> In Chapter 3 we have defined the aggregate supply function $Z = \varphi(N)$, which relates the employment N with the aggregate supply price of the corresponding output. The *employment function* only differs from the aggregate supply function in that it is, in effect, its inverse function and is defined in terms of the wage-unit; the object of the employment function being to relate the amount of the effective demand, measured in terms of the wage-unit, directed to a given firm or industry or to industry as a whole with the amount of employment, the supply price of the output of which will compare to that amount of effective demand. Thus, if an amount of effective demand D_{wr}, measured in wage-units, directed to a firm or industry calls forth an amount of employment N_r in that firm or industry, the employment function is given by $N_r = F_r(D_{wr})$. Or, more generally, if we are entitled to assume that D_{wr} is a unique function of the total effective demand D_w, the employment function is given by $N_r = F_r(D_w)$. (1936, p. 280)

Thus, if we assume the rates at which labor is demanded measured in wage-units in the investment- and consumption-goods industries are direct functions of the effective demands for investment ($\boldsymbol{I_t^{we}}$) and consumption ($\boldsymbol{C_t^{we}}$) goods, respectively, measured in wage-units in each of these industries—that is, are direct functions of the level of output at which producers *expect* to maximize their profits—we can write the demand for labor measured in wage-units in the investment-goods industries ($\boldsymbol{N_t^{wid}}$) as:

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net income is a unique function of gross income. See Keynes (1936, chaps. 4, 8).

$$(\mathbf{18}) \quad N_t^{wid} = n^{id}(I_t^{we}), \qquad n^{id'} > 0,$$

where the value of N_t^{wid} is expressed in terms of hours of ordinary labor per unit of time. Similarly, the demand for labor measured in wage-units in the consumption-goods industries (N_t^{wcd}) is given by:

$$(\mathbf{19}) \quad N_t^{wcd} = n^{cd}(C_t^{we}), \qquad n^{cd'} > 0,$$

and "if we are entitled to assume that [employment in each firm or industry] is a unique function of the total effective demand" (**18**) and (**19**) imply that Keynes' aggregate employment function can be written as: [8]

$$(\mathbf{20}) \quad \begin{aligned} N_t^w &= n^{id}(Y_t^{we}) + n^{cd}(Y_t^{we}) \\ &= n(Y_t^{we}), \qquad n' = 1, \end{aligned}$$

where Y_t^{we} is the aggregate effective demand—that is, the aggregate level of output at which producers *expect* to maximize their profits—as given by the sum of C_t^{we} and I_t^{we}:

$$(\mathbf{21}) \quad Y_t^{we} = C_t^{we} + I_t^{we}.$$

And since the "*employment function* only differs from the aggregate supply function in that it is, in effect, its inverse and is defined in terms of the wage-unit," the inverse of (**20**) yields Keynes' aggregate supply function defined in terms of wage-units:

$$(\mathbf{22}) \quad Y_t^{ws} = n^{-1}(N_t^w), \quad n^{-1'} = 1,$$

"which relates the employment [N_t^w] with the aggregate supply price [Y_t^{ws}] of the corresponding output [Y_t^w]" (= $Y_t^{we} = Y_t^{ws}$) measured in wage-units. (cf., Brady) And since output Y_t^w is determined by effective demand Y_t^{ed} (=Y_t^w) the

[8] Since the *aggregate* employment function is defined in terms of wage-units *net of user cost*, both N_t^w and Y_t^{we} define the number of hours-of-ordinary-labor/time-unit needed to satisfy Y_t^{we}. Thus, N_t^w = Y_t^{we} and dN_t^w = dY_t^{we} which implies that $n' = \mathbf{n}^{-1'} = 1$. See footnote 3 above and Keynes (1936, p. 55n).

inverse of Keynes' aggregate employment function (**20**) allows us to write Keynes' aggregate demand function (**17**) as:

$$(\mathbf{23}) \quad \begin{aligned} \boldsymbol{Y_t^{wd}} &= \boldsymbol{c}\left(\boldsymbol{n^{-1}}(\boldsymbol{N_t^w})\right) + \boldsymbol{i}\left(\boldsymbol{R_t}, \boldsymbol{P_t^a}, \boldsymbol{n^{-1}}(\boldsymbol{N_t^w})\right) \\ &= \boldsymbol{y^d}(\boldsymbol{N_t^w}, \boldsymbol{R_t}, \boldsymbol{P_t^a}), \quad \boldsymbol{y_1^d}, \boldsymbol{y_3^d} > 0, \ \boldsymbol{y_2^d} < 0, \end{aligned}$$

where $\boldsymbol{Y_t^{wd}}$ denotes the aggregate demand at each level of employment $\boldsymbol{N_t^w}$ both measured in wage-units given the rate of interest $\boldsymbol{R}$ and price of non-debt assets $\boldsymbol{P_t^a}$.

II. Dynamic Adjustment Functions

In specifying the dynamic adjustment functions that determine the behavior of the individual variables in Keynes' aggregate model it is assumed that demanders and suppliers of money adjust the rate of interest $\boldsymbol{R_t}$ to equate the demand for money $\boldsymbol{M_t^{wd}}$ (**2**) to the supply of money $\boldsymbol{M_t^w}$ (**3**) measured in wage-units in accordance with what Leijonhufvud referred to as Marshall's "laws of motion" (pp. 61-77):

$$(\mathbf{24}) \quad \begin{aligned} \boldsymbol{dR_t} &= \boldsymbol{g^r}\left(\boldsymbol{M_t^{wd}} - \boldsymbol{M_t^{ws}}\right) \\ &= \boldsymbol{g^r}\left(\boldsymbol{m^d}(\boldsymbol{Y_t^w}, \boldsymbol{R_t}) - \boldsymbol{m^s}(\boldsymbol{R_t})\right) \end{aligned}$$

$$(\mathbf{25}) \quad \begin{aligned} \boldsymbol{dM_t} &= \boldsymbol{g^r}\left(\boldsymbol{M_t^{wd}} - \boldsymbol{M_t^w}\right) \\ &= \boldsymbol{g^r}\left(\boldsymbol{m^d}(\boldsymbol{Y_t^w}, \boldsymbol{R_t}) - \boldsymbol{M_t^w}\right), \end{aligned}$$

where $\boldsymbol{dR_t}$ is the time derivative operator $\boldsymbol{d}$ ($\boldsymbol{=d/d\hat{t}}$) applied to $\boldsymbol{R_t}$, and the time derivative function $\boldsymbol{g^r}$ (as well as the time derivative functions specified below) is (are) assumed to increase monotonically through the origin.[9] It is

[9] It should be noted that the time derivative functions in this model are not assumed to be continuous, well-behaved mathematical functions even though for ease of exposition they will be discussed as such. Cf., Hayes, Brady, Lavoie and Godley, and Keynes (1936).

also assumed that demanders and suppliers of non-debt assets adjust the price of non-debt assets $\boldsymbol{P_t^a}$ to equate the demand for non-debt assets $\boldsymbol{A_t^{wd}}$ (**5**) to the existing stock of non-debt assets $\boldsymbol{A_t^{ws}}$ (**4**) measured in wage-units:

$$(\mathbf{26}) \quad \begin{aligned} \boldsymbol{dP_t^a} &= \boldsymbol{g^{pa}}\left(\boldsymbol{A_t^{wd} - A_t^{sw}}\right) \\ &= \boldsymbol{g^{pa}}\left(\boldsymbol{a^d}(\boldsymbol{P_t^a}, \boldsymbol{R_t}, \boldsymbol{Y_t^w}) - \boldsymbol{A_t^w}\right). \end{aligned}$$

Next it is assumed that producers in the investment- and consumption-goods industries adjust their expectations to equate the effective demands for consumption $\boldsymbol{C_t^{we}}$ and investment $\boldsymbol{I_t^{we}}$ goods to the actual demands for these goods as defined by the inverses of (**9**) and (**14**):

$$(\mathbf{27}) \quad \begin{aligned} \boldsymbol{dC_t^{we}} &= \boldsymbol{g^{ce}}\left(\boldsymbol{C_t^{wd} - C_t^{we}}\right) \\ &= \boldsymbol{g^{ce}}\left(\boldsymbol{c^{dp-1}}(\boldsymbol{P_t^c}, \boldsymbol{Y_t^w}) - \boldsymbol{C_t^{we}}\right) \end{aligned}$$

$$(\mathbf{28}) \quad \begin{aligned} \boldsymbol{dI_t^{we}} &= \boldsymbol{g^{ie}}\left(\boldsymbol{I_t^{wd} - I_t^{we}}\right) \\ &= \boldsymbol{g^{ie}}\left(\boldsymbol{i^{dp-1}}\left(\boldsymbol{P_t^i}, \boldsymbol{R_t}, \boldsymbol{P_t^a}, \boldsymbol{Y_t^w}\right) - \boldsymbol{I_t^{we}}\right) \end{aligned}$$

as they adjust the rates of consumption $\boldsymbol{C_t^w}$ and investment $\boldsymbol{I_t^w}$ goods production to their respective effective demands:

$$(\mathbf{29}) \quad \boldsymbol{dC_t^w} = \boldsymbol{g^c}(\boldsymbol{C_t^{we} - C_t^w})$$

$$(\mathbf{30}) \quad \boldsymbol{dI_t^w} = \boldsymbol{g^i}(\boldsymbol{I_t^{we} - I_t^w})$$

and that suppliers and demanders in the markets for investment and consumption goods adjust the prices of investment $\boldsymbol{P_t^i}$ and consumption $\boldsymbol{P_t^c}$ goods to equate the supplies ($\boldsymbol{I_t^s}$), ($\boldsymbol{C_t^s}$) and demands ($\boldsymbol{I_t^d}$), ($\boldsymbol{C_t^d}$) for investment and consumption goods as given by the inverses of the supply prices $\boldsymbol{P_t^{is}}$ (**13**), $\boldsymbol{P_t^{cs}}$ (**8**) and demand prices $\boldsymbol{P_t^{id}}$(**14**), $\boldsymbol{P_t^{cd}}$ (**9**) for these goods:

$$(\mathbf{31}) \quad \begin{aligned} \boldsymbol{dP_t^i} &= \boldsymbol{g^{pi}}\left(\boldsymbol{I_t^d - I_t^s}\right) \\ &= \boldsymbol{g^{pi}}\left(\boldsymbol{i^{dp-1}}\left(\boldsymbol{P_t^i}, \boldsymbol{R_t}, \boldsymbol{P_t^a}, \boldsymbol{Y_t^w}\right) - \boldsymbol{i^{sp-1}}\left(\boldsymbol{P_t^i}\right)\right) \end{aligned}$$

$$(\mathbf{32}) \quad \begin{aligned} \boldsymbol{dP_t^c} &= \boldsymbol{g^{pc}}\left(\boldsymbol{C_t^d - C_t^s}\right) \\ &= \boldsymbol{g^{pc}}\left(\boldsymbol{c^{dp-1}}(\boldsymbol{P_t^c}, \boldsymbol{Y_t^w}) - \boldsymbol{c^{sp-1}}(\boldsymbol{P_t^c})\right). \end{aligned}$$

Finally, it is assumed that producers adjust aggregate employment ($\boldsymbol{N_t^w}$) to equate the aggregate supply of output $\boldsymbol{Y_t^{ws}}$ (**22**) and the aggregate effective demand for output $\boldsymbol{Y_t^{we}}$ (**21**): [10]

$$(\mathbf{33}) \quad \boldsymbol{dN_t^w = g^{nd}(Y_t^{we} - Y_t^{ws})} \\ \boldsymbol{= g^{nd}\left(Y_t^{we} - n^{-1}(N_t^w)\right)}$$

as the effective demand for output $\boldsymbol{Y_t^{we}}$ (**21**) adjusts to the actual demand for output $\boldsymbol{Y_t^{wd}}$ (**23**) by way of the identity implied by (**27**) and (**28**):

$$(\mathbf{34}) \quad \boldsymbol{dY_t^{we} = dC_t^{we} + dI_t^{we}} \\ \boldsymbol{= g^{ce}\left(c^{dp-1}\left(P_t^{cd}, Y_t^w\right) - C_t^{we}\right) + g^{ie}\left(i^{dp-1}\left(P_t^{id}, R_t, P_t^a, Y_t^w\right) - I_t^{we}\right),}$$

and aggregate output/income $\boldsymbol{Y_t^w}$ adjusts by way of the identity implied by (**29**) and (**30**):

$$(\mathbf{35}) \quad \boldsymbol{dY_t^w = dC_t^w + dI_t^w} \\ \boldsymbol{= g^c(C_t^{we} - C_t^w) + g^i(I_t^{we} - I_t^w).}$$

III. Structure of Keynes' Aggregate Model

The adjustment functions (**24**) - (**35**) define the way in which changes in twelve endogenous variables are determined in Keynes' aggregate model: $\boldsymbol{R_t}$, $\boldsymbol{M_t^w}$, $\boldsymbol{P_t^a}$, $\boldsymbol{P_t^i}$, $\boldsymbol{P_t^c}$, $\boldsymbol{C_t^w}$, $\boldsymbol{C_t^{we}}$, $\boldsymbol{I_t^w}$, $\boldsymbol{I_t^{we}}$, $\boldsymbol{Y_t^{ew}}$, $\boldsymbol{N_t^w}$, and $\boldsymbol{Y_t^w}$. Since these functions are assumed to pass through the origin the system is in equilibrium in the sense that there is no reason for any variable to change when all of the adjustment functions are equal to zero. This gives us twelve equilibrium conditions which contain twelve endogenous variables as summarized in **Table 1**.

[10] Cf., Keynes: "the effects on employment of the realised sale-proceeds of recent output and those of the sale-proceeds expected from current input; and producers' forecasts are more often gradually modified in the light of results than in anticipation of prospective changes" (1936, p. 51).

Table 1: Structure of Keynes' Aggregate Model		
Market	**Equilibrium Conditions**	**Endogenous Variables**
Assets	$M_t^{ws} = M_t^w$ | $M_t^{ws} = M_t^{wd}$ $A_t^{wd} = A_t^w$	M_t^w, R_t, P_t^a
Investment	$I_t^{we} = I_t^{wd}$ | $I_t^{we} = I_t^w$ $P_t^{id} = P_t^{is}$	I_t^{we}, I_t^w, P_t^i
Consumption	$C_t^{we} = C_t^{wd}$ | $C_t^{we} = C_t^w$ $P_t^{cd} = P_t^{cs}$	C_t^{we}, C_t^w, P_t^c
Labor	$Y_t^{ws} = Y_t^{we}$	N_t^w
Identities	$Y_t^{we} = C_t^{we} + I_t^{we}$ $Y_t^w = C_t^w + I_t^w$	Y_t^{we}, Y_t^w

This table outlines the mathematical structure of the short-run aggregate model described by Keynes in the passage quoted above. The equilibrium values of the endogenous variables are assumed to be determined by the behavioral relationships defined by the aggregate behavioral equations (**1**) - (**23**) *given the assumption that employment is determined by the effective demands of producers as expectations adjust to equate effective demands and other variables to the actual demands that exist in markets by way of the adjustment functions (**24**) - (**35**).*

The fundamental difference between the structure of this model and that of the Walrasian paradigm of neoclassical economics is that Keynes' behavioral equations are assumed to be consistent with *Marshallian* supply and demand functions rather than the *Walrasian* supply and demand functions assumed by neoclassical economists. (Brady; Clower; Hayes; Leijonhufvud) They are presumed to be determined by the optimizing behavior of decision-making units as they interact in markets, just as Walrasian supply and demand functions are presumed to be determined by optimizing behavior. The difference is that in Keynes' understanding of these functions they are specified by isolating those factors that are perceived to have a *direct* effect on the *willingness* of buyers and sellers to buy and sell in individual markets wheth-

er the system as a whole is in equilibrium or not without assuming that these choices are constrained by an arbitrary Walrasian budget constraint. (Blackford 2020a, pp. 20-77; 2019a) Instead, they are derived by observing the actual behavior of decision-making units in markets, hypothesizing with regard to the motivations of these units given their *expectations* with regard to those magnitudes that affect their choices *directly* in each individual market, and then reasoning through the logical implications of what the actual choices available to decision-making units and their motivations and expectations imply with regard to their willingness to buy and sell in individual markets. (cf., Blackford 1975; 1976; Clower; Lavoie and Godley) As a result, even though the set of equilibrium conditions specified above when taken together define a general equilibrium of the system as a whole they are the product of a *partial* equilibrium analysis of individual markets rather than the product of a *general* equilibrium analysis as such.[11]

It should also be noted that the supply and demand for loanable funds are not independent behavioral functions in this model. In a world in which decision-making units do not go into debt simply for the sake being in debt—*a world in which debt, in itself, offers no satisfaction or utility*—there must be a demand for *money* for some reason other than the satisfaction of being in debt *before* there can be a willingness to borrow money. Thus, it is assumed that decision-making units borrow *money* only to meet their financial needs for *money*. Similarly, it is assumed that decision-making units lend money only to dispose of excess money balances they have no use for otherwise or in the case of trade credit to facilitate current transactions by providing for the payment of *money* at a later date. This makes the supply and demand for loanable funds *ex post* functions, determined within the system by

[11] See Marshall (1920; 1961, Books III-IV) and cf., Keynes (1936, Books III-IV), Blackford (2020a), Hayes, Brady, and Lavoie and Godley.

the supply and demand for money as dictated by the desired transactions of decision-making units as determined by the behavioral equations of the model. As a result, there are no redundant equations in the model outlined in **Table 1**, and Walras' Law—*a law that can only be enforced by a mythical auctioneer*—has no relevance in this model, just as it has no relevance in Keynes' general theory or in the real world. (Blackford 1975; 1976; 2020a, pp. 20-77; 2019a; 2019c; Clower)[12]

IV. Short-Run Equilibrium

The way in which the short-run equilibrium values of the variables in the model are determined by supply and demand in individual markets and the ways in which the interconnections between markets are summarized by Keynes' consumption function (**11**), MEC schedule (**16**), and the asset market equilibrium function (**7**) are illustrated in **Figure 1** where:

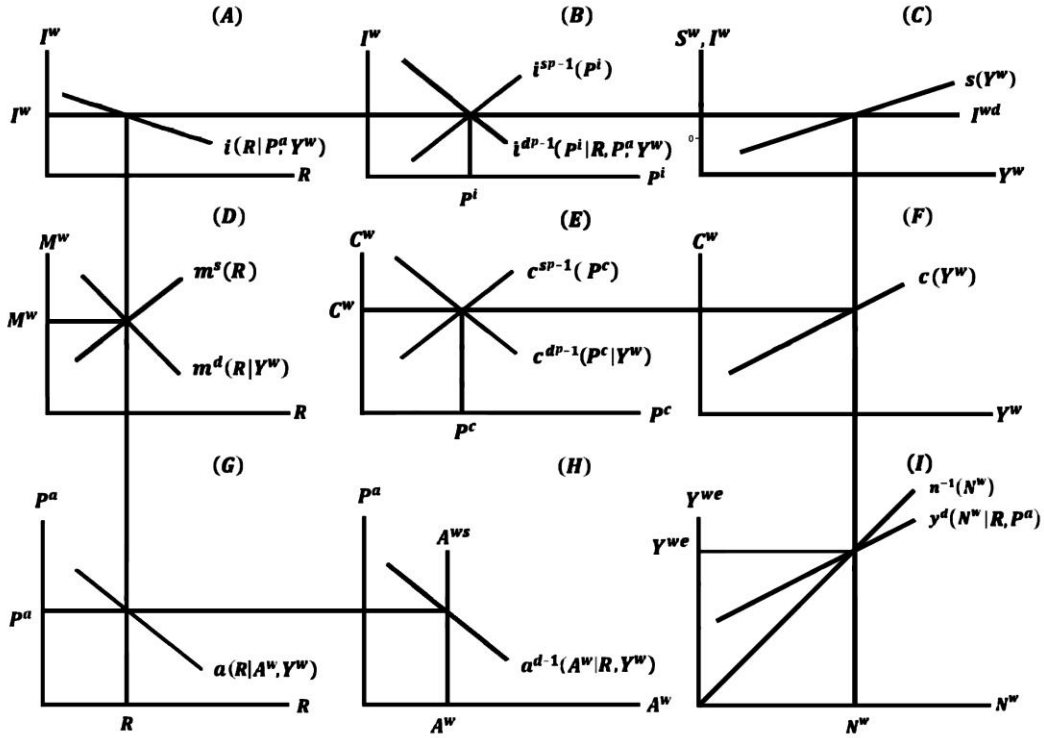
1. Given the rate of interest $\boldsymbol{R}$, the price of non-debt assets $\boldsymbol{P^a}$, and output/income $\boldsymbol{Y^w}$ the equilibrium price $\boldsymbol{P^i}$ and rate of investment goods production $\boldsymbol{I^w}$ are determined in panels (**A**) and (**B**) by the demanders and suppliers of investment goods as dictated by the inverses of the demand price $\boldsymbol{i^{dp-1}(P^i\,|R, P^a, Y_t^w)}$ (**14**) and supply price $\boldsymbol{i^{sp-1}(P^i)}$ (**13**) of investment goods functions in panel (**B**) from which Keynes' MEC schedule $\boldsymbol{i(R|P^a, Y^w)}$ (**16**) in panel (**A**) is derived.
2. Given the equilibrium rate of investment $\boldsymbol{I^w}$ the value of output/income

[12] Cf., Lavoie and Godley. This does not mean that the system-wide consistency requirements that Lavoie and Godley (p. 14) examined are violated. It only means that there is no reason to believe that at any given *point in time* the excess demands in the model sum to zero. As they have noted "the λ parameters will be shifting around like mad, as people change opinions on what is appropriate, but they are always subject to the adding-up constraints." (p. 144)

$\boldsymbol{Y^w}$ is determined by savers and investors in accordance with Keynes' savings function $\boldsymbol{s(Y^w)}$ (**12**) in panel (**C**) which is given by one minus Keynes' consumption function $\boldsymbol{c(Y^w)}$ (**11**) in panel (**F**).

3. Given output/income $\boldsymbol{Y^w}$ the equilibrium rate of interest $\boldsymbol{R}$ is determine by the demanders and suppliers of money as dictated by the demand $\boldsymbol{m^d(R|Y^w)}$ (**2**) and supply $\boldsymbol{m^s(R)}$ (**3**) of money functions in panel (**D**), and the equilibrium price $\boldsymbol{P^c}$ and rate of consumption goods production $\boldsymbol{C^w}$ are determined in panels (**E**) and (**F**) by the demanders and suppliers of consumption goods as dictated by the inverses of the demand price $\boldsymbol{c^{dp-1}(P^c|Y^w)}$ (**9**) and supply price $\boldsymbol{c^{sp-1}(P^c)}$ (**8**) of consumption goods functions in panel (**E**) from which Keynes' consumption function $\boldsymbol{c(Y^w)}$ (**11**) in panel (**F**) is derived.

Figure 1: Short-Run Equilibrium.



4. Given the rate of interest $\boldsymbol{R}$ and level of output/income $\boldsymbol{Y^w}$ the equilibrium price of non-debt assets $\boldsymbol{P^a}$ is determine in panels (**G**) and (**H**) by the de-

manders and suppliers of non-debt assets as dictated by the supply of non-debt assets $\boldsymbol{A^{ws}}$ (**4**) and the inverse of the demand for non-debt assets $\boldsymbol{a^{d-1}(A^w|R,Y^w)}$ (**5**) function in panel (**H**) from which the non-debt asset equilibrium function $\boldsymbol{a(R|P^a,Y^w)}$ (**7**) in panel (**G**) is derived.

5. Given the rate of interest $\boldsymbol{R}$ and the price of non-debt assets $\boldsymbol{P_t^a}$ the point of effective demand $\boldsymbol{Y^{we}}$ and equilibrium rate of employment $\boldsymbol{N^w}$are determined by producers at the intersection of Keynes' aggregate supply $\boldsymbol{n^{-1}(N^w)}$ (**22**) and demand $\boldsymbol{y^d(N^w|R,P^a)}$ (**23**) schedules in panel (**I**). [13]

But what is most significant about the model embodied in equations (**1**) through (**35**) above is that it formalizes the analytical framework develop by Keynes throughout *The General Theory*—a framework within which a *causal analysis of* the *dynamic behavior of the economic system is possible.*

V. Changes in Short-Run Equlibrium

As noted above, rather than view the economic system from the perspective of a set of Walrasian equations Keynes viewed the system from the perspective of a set of Marshallian partial equilibrium models in which *the values of individual variables are determined by the choices of those decision-making units that actually have the power to determine the value of each variable at each point in time as the system evolves through time*. (Blackford 2020a, pp. 18-73; 2019a; 2019b; 2019c; 2019d) Accordingly, in examining the dynamic behavior of the model specified above it is assumed that given the

[13] For a detailed discussion of the way in which equilibrium is defined and achieved in the works of Marshall, Keynes, and neoclassical economists see Hayes (2006), Kregel, and Lavoie and Godley. The focus of this paper is on what Hayes refers to as "system" equilibrium and "the dynamic process of convergence in which a series of positions of short-period equilibrium trace a path towards a position of long-period equilibrium" (2006, pp. 3-9). See also Blackford (2020a; 2021).

money wage W_t, stock non-debt assets A_t^w, and other exogenous variables and parameters of the model:

1. The prices and rates of production and sale of goods and resources along with the price of non-debt assets (i.e., the complex of prices on real and financial non-debt assets) is (are) determined through the interactions of suppliers and demanders in the markets for goods, resources, and non-debt assets.
2. The rate of interest (i.e., the complex of rates of interest on new loans and debt assets) is (are) determined by the suppliers and demanders for money (i.e., liquidity) in the money market.
3. Employment is determined by producers in accordance with their *effective demands*—that is, the point at which producers *expect* to maximize their profits.
4. Income (i.e., the value of output produced) is determined by savers and investors as they interact in the markets for consumption and investment goods.
5. And the entire process by which these variables are determined *at each point in time* is governed by the *expectations* of decision-making units as their expectations adjust to the realized results that are achieved within the system as the system evolves *through time*.

These assumptions made it possible for Keynes to isolate those factors that *directly* and *in themselves* determine each variable at *each point in time* whether the *system* is in equilibrium or not. This makes it possible to establish *the temporal order in which events must occur* as decision-making units respond to changes in the exogenous determinants of the variables *in each sector* of the economy. It is the ability to establish the temporal order in which events must occur that makes it possible to separate cause and effect, and it is the ability to separate cause and effect that makes a *logically con-*

sistent, causal analysis of the dynamic behavior possible in Keynes' general theory and in economics in general.[14]

Consider, for example, an increase in thriftiness that takes the form of an increase in the demand for securities—that is, an increase in the propensity to consume (**16**) that takes the form of an increase in the demand for non-debt assets (**5**). How will this affect the short-run equilibrium position of the economic system shown in **Figure 1**, and *how will the new short-run equilibrium come about?*

The *direct* effects will be to *increase* the saving function $\boldsymbol{s}(\boldsymbol{Y^w})$ (**12**) in panel (**C**) of **Figure 1**, the demand for non-debt assets $\boldsymbol{a^{d-1}}(\boldsymbol{A^w}|\boldsymbol{R},\boldsymbol{Y^w})$ (**5**) in panel (**H**), and the non-debt asset equilibrium function $\boldsymbol{a}(\boldsymbol{R}|\boldsymbol{A^w})$ (**7**) in panel (**G**) and to *decrease* the consumption function $\boldsymbol{c}(\boldsymbol{Y^w})$ (**11**) in panel (**F**) and the demand for consumption goods $\boldsymbol{c^{dp-1}}(\boldsymbol{P^c}|\boldsymbol{Y^w})$ (**9**) in panel (**E**); at the same time the transfer of a portion of the revolving fund of money in circulation from the purchase of consumption goods to the purchase of non-debt assets will lead to an increase in the demand for money $\boldsymbol{m^d}(\boldsymbol{R}|\boldsymbol{Y^w})$ (**2**) in panel (**D**) since there will be no reduction of the demand for transactions balances in the consumption goods industries until employment and output fall in those industries. (Blackford 2019b; 2019c; 2019d) Thus, the direct effects will be to create an excess supply in the market for consumption goods in panel (**E**) and excess demands for money in panel (**D**) and non-debt assets in panel (**H**).

The excess demand for money will, in turn, *cause* the rate of interest $\boldsymbol{R}$ to *increase* in accordance with (**24**) which will have negative effects on the increase in the demand for non-debt assets $\boldsymbol{a^{d-1}}(\boldsymbol{A^w}|\boldsymbol{R},\boldsymbol{Y^w})$ in panel (**G**), the

[14] See Blackford (2020a, pp. 20-77; 2019a; 2019b; 2019c), Hume, and cf., Brady, Hayes, and Lavoie and Godley.

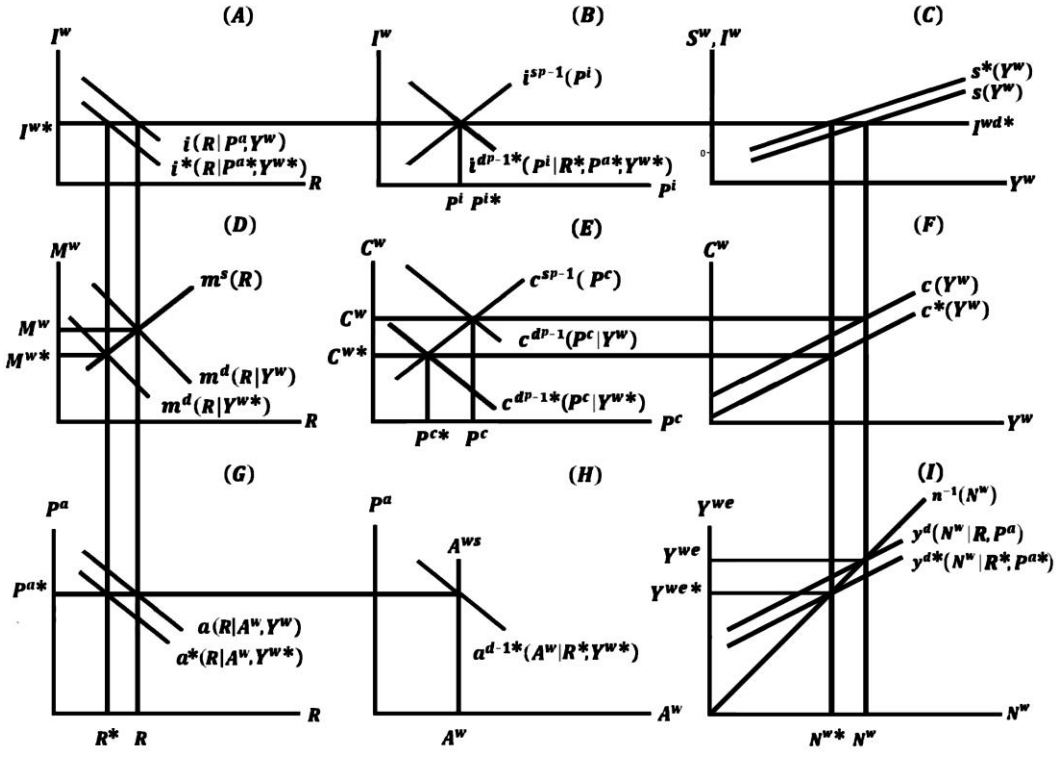
demand for investment $\boldsymbol{i^{dp-1}(P^i\,|R,P^a,Y_t^w)}$ in panel (**B**), and aggregate demand $\boldsymbol{y^d(N^w|R,P^a)}$ in panel (**I**). At the same time, as producers in the consumption goods industries respond to the excess supply for consumption goods and adjust their effective demands for consumption goods $\boldsymbol{C^{we}}$ to the actual demands for consumption goods $\boldsymbol{C^{wd}}$ in accordance with (**27**), the price $\boldsymbol{P^c}$ and production $\boldsymbol{C^w}$ of consumption goods in panel (**E**) will begin to fall in accordance with (**32**) and (**29**) with a concomitant fall in aggregate demand $\boldsymbol{y^d(N^w|R,P^a)}$ in panel (**I**) as the effective demand for output $\boldsymbol{Y^{we}}$ adjusts to the actual demand for output $\boldsymbol{Y^{wd}}$ in accordance with (**34**). This will lead to a fall in employment $\boldsymbol{N^w}$ and output/income $\boldsymbol{Y^w}$ in accordance with (**33**) and (**35**).

The fall in output/income $\boldsymbol{Y^w}$ will, in turn, have negative effects on the demands for money $\boldsymbol{m^d(R|Y^w)}$ in panel (**D**), investment $\boldsymbol{i^{dp-1}(P^i\,|R,P^a,Y_t^w)}$ in panel (**B**), consumption $\boldsymbol{c^{dp-1}(P^c|Y^w)}$ in panel (**D**), and non-debt assets $\boldsymbol{a^{d-1}(A^w|R,Y^w)}$ in panel (**G**). The resulting excess demand for money in panel (**D**) will cause the rate of interest $\boldsymbol{R}$ to fall in accordance with (**24**) which will have positive effects on the demands for investment goods $\boldsymbol{i^{dp-1}(P^i|R,P^a,Y_t^w)}$ in panel (**B**), non-debt assets $\boldsymbol{a^{d-1}(A^w|R,Y^w)}$ in panel (**G**), and aggregate demand $\boldsymbol{y^d(N^w|R,P^a)}$ in panel (**I**). At the same time, the decrease in the value of aggregate output/income $\boldsymbol{Y^w}$ that *results* from the decrease in the output of consumption goods $\boldsymbol{C^w}$ will have the effect of further decreasing the demand for consumption goods $\boldsymbol{c^{dp-1}(P^c|Y^w)}$ in panel (**E**). This, in turn, will set in motion a *causal* feedback loop within the system (the multiplier) as the excess supply of consumption goods that results *causes* a decrease in the price of consumption goods $\boldsymbol{P^c}$ in panel (**E**) in accordance with (**29**), and as the effective demand for consumption goods $\boldsymbol{C_t^{we}}$ adjusts to the actual demand for consumption goods $\boldsymbol{C_t^{wd}}$ in accordance with (**27**) producers will decrease the output of consumptions goods $\boldsymbol{C^w}$ in accordance with

(**29**). This will further decrease the aggregate demand $\boldsymbol{y^d(N^w|R,P^a)}$ in panel (**I**) which will *cause* a further decrease aggregate effective demand $\boldsymbol{Y^{we}}$, aggregate employment $\boldsymbol{N^w}$, and output/income $\boldsymbol{Y^w}$ in accordance with (**34**), (**33**), and (**35**), respectively, which will further decrease the demand for consumption goods $\boldsymbol{c^{dp-1}(P^c|Y^w)}$ in panel (**E**).

The further decrease in aggregate output/income $\boldsymbol{Y^w}$ *caused* by the further decrease in consumption goods produced $\boldsymbol{C^w}$ will *cause* a further decrease in the price $\boldsymbol{P^c}$ and output $\boldsymbol{C^w}$ of consumption goods which will *cause* a further decrease in employment $\boldsymbol{N^w}$, output/income $\boldsymbol{Y^w}$, and the demand for money $\boldsymbol{m^d(R|Y^w)}$ which will *cause* a further decrease in the rate of interest $\boldsymbol{R}$. The fall in output/income $\boldsymbol{Y^w}$ will enhance the negative effects on the demands for investment goods $\boldsymbol{i^{dp-1}(P^i|R,P^a,Y_t^w)}$ and non-debt assets $\boldsymbol{a^{d-1}(A^w|R,Y^w)}$ while the resulting fall in the rate of interest $\boldsymbol{R}$ will further enhance the positive effects on these demands.

Figure 2: An Increase in Thriftiness.



This *causal* loop must continue until system has achieved the short-run system equilibrium depicted in **Figure 2**, where the rate of interest, consumption, employment, the price of consumption goods, and output/income have decreased from $\boldsymbol{R}$, $\boldsymbol{N^w}$, $\boldsymbol{P^c}$, $\boldsymbol{C^w}$, and $\boldsymbol{Y^w}$ to $\boldsymbol{R^*}$, $\boldsymbol{N^w*}$, $\boldsymbol{P^c*}$, $\boldsymbol{C^w*}$, and $\boldsymbol{Y^w*}$. The prices of non-debt assets $\boldsymbol{P^a}$ and investment goods $\boldsymbol{P^i}$ along with the level of investment $\boldsymbol{I^w}$ are left unchanged in this figure since the direction of change of these variables is indeterminate; whether these variables increase or decrease depends on the relative strengths of the positive effects of the fall in the rate of interest $\boldsymbol{R}$ on the demand for investment goods $\boldsymbol{i^{dp-1}(P^i|R, P^a, Y_t^w)}$ in panel (**B**) and the demand for non-debt assets $\boldsymbol{a^d(P^a|R, Y_t^w)}$ in panel (**H**) and the negative effects of the fall in output/income $\boldsymbol{Y^w}$ on these two demands. (Blackford 2019b)

VI. Achieving Long-Run Equilibrium

In specifying Keynes' short-run equilibrium model above the stock of non-debt assets $\boldsymbol{A_t^w}$ is assumed to be exogenous, but positive net investment must, by definition, increase $\boldsymbol{A_t^w}$ over time. While the effects of this increase can be assumed to be insignificant in the short run, Keynes argued that these effects can be dramatic in the long run. Specifically, he argued that the "position of [long-run] equilibrium, under conditions of *laissez-faire*, will be one in which employment is low enough and the standard of life sufficiently miserable to bring savings to zero." (1936, p. 217-18)

The nature of the long-run equilibrium envisioned by Keynes can be demonstrated in the model specified above by noting that "an increased investment in any given type of capital during any period of time the marginal efficiency of that type of capital will diminish as the investment in it is increased" (Keynes, 1936, p. 136) and explicitly including the stock of non-debt assets $\boldsymbol{A_t^w}$ in the demand price of investment function (**14**) to obtain:

$$(\boldsymbol{14a}) \quad \boldsymbol{P_t^{id} = i^{dp}(I_t^w, R_t, A_t^w, P_t^a, Y_t^w)}, \qquad \boldsymbol{i_3^{dp}, i_5^{dp} > 0, \; i_1^{dp}, i_2^{dp}, i_3^{dp} < 0.}$$

This implies that the MEC (**16**) and aggregate demand (**23**) functions become:

$$(\mathbf{16a})\quad I_t^{wd} = i(R_t, A_t^w, P_t^a, Y^w), \qquad i_3^d, i_4^d > 0,\ i_1^d,\ i_2^d < 0$$

$$(\mathbf{23a})\quad Y_t^{wd} = c\left(n^{-1}(N_t^w)\right) + i\left(R_t, A_t^w, P_t^a, n^{-1}(N_t^w)\right)$$
$$= y^d(N_t^w, R_t, A_t^w, P_t^a), \qquad y_1^d, y_4^d > 0,\ y_2^d, y_3^d < 0,$$

and if, for ease of exposition, we assume constant returns to scale and a fixed money wage W_t the consumption (**8**) and investment (**13**) goods supply-price functions can be replaced with their long-run counterparts:

$$(\mathbf{8a})\qquad P_t^{cs} = P^{c*}$$

$$(\mathbf{13a})\qquad P_t^{is} = P^{i*}.$$

Finally, the adjustment equations (**28**) and (**34**) must be modified accordingly:

$$(\mathbf{28a})\quad dI_t^{we} = g^{ie}\left(I_t^{wd} - I_t^{we}\right)$$
$$= g^{ie}\left(i^{dp-1}\left(P_t^i, R_t, A_t^w, P_t^a, Y_t^w\right) - I_t^{we}\right)$$

$$(\mathbf{34a})\quad dY_t^{we} = dC_t^{we} + dI_t^{we}$$
$$= g^{ce}\left(c^{dp-1}(P_t^c, Y_t^w) - C_t^{we}\right) + g^{ie}\left(i^{dp-1}\left(P_t^i, R_t, A_t^w, P_t^a, Y_t^w\right) - I_t^{we}\right).$$

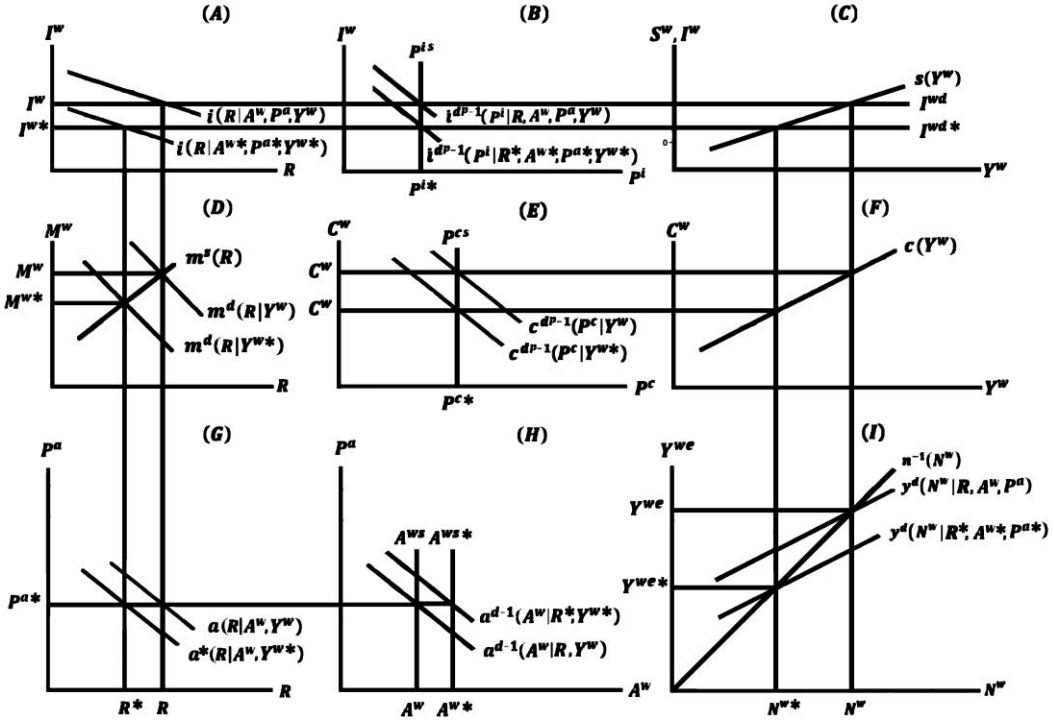
Given these extensions of the model specified above the mechanisms by which an increase in the capital stock leads to Keynes' long-run equilibrium can be demonstrated by examining the effects of an increase in the stock of non-debt assets from A^w to A^{w*} in panel (**H**) of **Figure 3**.

The direct effects of an increase in non-debt assets A^w will be to increase the supply of non-debt assets A^{ws} (**4**) in panel (**H**) and the non-debt asset equilibrium curve $a(R|A^w, Y^w)$ (**7**) in panel (**G**) as it decreases the demand for investment goods $i^{dp-1}(P^i|R, A^w, P^a, Y^w)$ (**14a**) in panel (**B**), the MEC curve $i(R|A^w, P^a, Y^w)$ (**16a**) in panel (**A**), and the aggregate demand curve $y^{wd}(N^w|R, A^w, P^a)$ (**23a**) in panel (**I**). The result will be excess supplies for non-debt assets in panel (**G**), investment goods in panel (**B**), and effective

demand in panel (**I**).

The excess supply for non-debt assets will cause the price of non-debt assets P^a to fall in accordance with (**26**), and as the effective demand for investment goods I_t^{we} adjusts to the actual demand for investment goods I_t^{wd} in accordance with (**28a**) the output of investment goods I_t^w will fall in accordance with (**30**). At the same time, as the aggregate effective demand for output Y_t^{we} adjusts to the actual demand for output Y_t^{wd} in accordance with (**34a**) aggregate employment N^w and output/income Y^w will fall in accordance with (**33**) and (**35**).

Figure 3: Toward Long-Run Equilibrium.



The fall in output/income Y^w will, in turn, decrease the transactions demand for money which will cause a fall in the demand for money $m^{wd}(R|Y^w)$ in panel (**D**). The resulting excess supply of money will cause a fall in the rate of interest R in accordance with (**25**) which will enhance the demand for non-debt assets $a^{d-1}(A^w|R,Y^w)$ in panel (*H*), the demand for investment

goods $\boldsymbol{i^{dp-1}(P^i|R, A^w, P^a, Y^w)}$ in panel (**B**), the MEC curve $\boldsymbol{i(R|A^w, P^a, Y^w)}$ (**16a**) in panel (**A**), and aggregate demand curve $\boldsymbol{y^{wd}(N^w|R, A^w, P^a)}$ in panel (**I**). At the same time, the fall in output/income $\boldsymbol{Y^w}$ will also cause a fall in the demand for consumption goods $\boldsymbol{c^{wd-1}(P^c|Y^w)}$ in panel (**E**). As the effective demand for consumption goods $\boldsymbol{C^{we}}$ adjusts to the actual demand for consumption goods $\boldsymbol{C^{wd}}$ in accordance with (**27**) the output of consumption goods $\boldsymbol{C^w}$ will fall in accordance with (**29**). This, in turn, will cause a further fall in output and income $\boldsymbol{Y^w}$ as the multiplier process moves the system to its new point of short-run equilibrium at $\boldsymbol{A^{w*}}$, $\boldsymbol{P^{a*}}$, $\boldsymbol{C^{w*}}$, $\boldsymbol{P^{cl}}$, $\boldsymbol{I^{w*}}$, $\boldsymbol{P^{il}}$, $\boldsymbol{N^{w*}}$, $\boldsymbol{Y^{w*}}$, and $\boldsymbol{R^*}$ in **Figure 3**. The price of non-debt assets $\boldsymbol{P^{a*}}$ is left unchanged in this figure since the direction of change of this variables is indeterminate; whether this variable increases or decreases depends on the relative strengths of the negative effects of the fall in output/income and the positive effects of the fall in the rate of interest $\boldsymbol{R}$ on the demand for non-debt assets $\boldsymbol{a^{d-1}(A^w|R, Y^w)}$ in panel (**H**) and, hence, is indeterminate.[15]

The stock of non-debt assets $\boldsymbol{A^w}$ must continue to increase, and the levels of investment $\boldsymbol{I^w}$, consumption $\boldsymbol{C^w}$, employment $\boldsymbol{N^w}$, output/income $\boldsymbol{Y^w}$, and the rate of interest **R** must continue to fall until the investment goods demand function $\boldsymbol{i^{dp-1}(p^i|R, A^w, P^a, Y^w)}$ in panel (**B**) has fallen to the point at

[15] It should be noted that even though the positive effects of the fall in the rate of interest $\boldsymbol{R}$ on the demand for investment goods $\boldsymbol{i^{dp-1}(P^i|R, A^w, P^a, Y^w)}$ in panel (**B**) inhibit the negative effects of the increase in non-debt assets $\boldsymbol{A^w}$ it is not assumed that level of investment $\boldsymbol{I_t^w}$ is indeterminate since the rate of interest $\boldsymbol{R}$ can only fall in this *ceteris paribus* situation *as a result* of the fall in output/income $\boldsymbol{Y^w}$ that is *caused* by a fall in investment $\boldsymbol{I^w}$; a fall in the rate of interest $\boldsymbol{R}$ *caused* by a fall in output/income $\boldsymbol{Y^w}$ cannot offset the fall in investment $\boldsymbol{I^w}$ that *caused* the fall in output/income $\boldsymbol{Y^w}$ that *caused* the fall in the rate of interest $\boldsymbol{R}$.

which investment $\boldsymbol{I^w}$ is just sufficient to replace the capital that is consumed in the process of producing $\boldsymbol{Y^w}$. At that point the long-run equilibrium will have been achieved as *net* investment will be zero and the stock of non-debt assets $\boldsymbol{A^w}$ will no longer increase.[16]

VII. Concluding Observations

The fact that the exogenous variables and parameters are assumed to be given in the analysis above does not mean that these factors cannot or will not change. It only means that the effects of changes in these factors can either be ignored in considering the problem at hand or that they are not determined in a systimatic way within the system such that their effects must be examined separately.

One exogenous factor that is particularly relavent to the history of economic controversy is the effects of a change in the supply of money $\boldsymbol{m^s(R)}$ with regard to the short-run equilibrium examined in **Figure 2**. Dennis H. Robertson argued that **Figure 2** exemplifies "a situation calling for a progressive increase in the supply of money" (1936, p. 8). This can be represented in **Figure 2** by a shift in the money supply function $\boldsymbol{m^s(R^*)}$ in panel (**D**) to the right which will create an excess supply of money that leads to a fall in the rate of interest **R** in accordance with (**24**). The resulting fall in **R** will increase the demand for non-debt assets $\boldsymbol{a^{d-1}*(A^w|R^*,Y^w*)}$ in panel (**H**) and the demand for investmen goods $\boldsymbol{i^{dp-1}*(P^i|R^*,P^a*,Y^w*)}$ in panel

[16] See Keynes (1936, pp. 27-32, 136, 211-15, 217-8, 228-31) and Blackford (2021). These results illustrate Keynes' paradox of thrift and are contrary to those obtained in Lavoie and Godley (LG) (pp. 117, 363-4). The fundamental difference between the model specified above and that of LG is that the LG model assumes households target a specific wealth to income ratio that increases consumption as non-debt assets increase. This is an empirical issue that is beyond the scope of this paper.

(**B**). The resulting excess demands will cause an increase in a) the price of non-debt assets $\boldsymbol{P^a}$ in panel (**H**), b) the price $\boldsymbol{P^i}$ and output $\boldsymbol{I^w}$ of investment goods in panel (**B**), and c) in employment $\boldsymbol{N^w}$ and output/income $\boldsymbol{Y^w}$ in panel (**I**) as the effective demands for investment goods $\boldsymbol{I^{we}}$ and output $\boldsymbol{Y^{we}}$ adjust to the actual demands for investment goods $\boldsymbol{I^{wd}}$ and output $\boldsymbol{Y^{wd}}$ in accordance with (**26**), (**30**), (**33**), (**35**), (**28a**) and (**34**), respectively.

The increase in output/income $\boldsymbol{Y^w}$ will cause an increase in the quantity of consumption goods demanded $\boldsymbol{C^{wd}}$ which will initiate a feedback loop that will enhance the increases in employment $\boldsymbol{N^w}$ and output/income $\boldsymbol{Y^w}$ as the supply of money continues to shift to the right until either the system reaches full employment, or the rate of interest is forced to zero after adjusting for the cost of bringing browers and lenders together. Once the rate of interest is forced to zero employment $\boldsymbol{N^w}$ and output/income $\boldsymbol{Y^w}$ must begin to fall until the level of net investment is forced to zero. In the end, as the system aproaches its long-run equilibrium in **Figure 3** the limiting factor becomes the point at which the level of net investment is forced to zero.

A second exogenous factor that is particularly relevant is the effects of a change in the money wage $\boldsymbol{W}$. Since Keynes undertook a detailed examination of the various ways in which changes in the money wage can affect the system in Chapter 19 of *The General Theory*, all of which can be examined within the context of the model specified above (Blackford 2020b, pp. 211-18), there is no need to repeat Keynes' analysis here save to note that, in the end, Keynes concluded that wage flexibility affects the stability of the system rather than the level of employment.

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Appendix: List of Equations

I. Behavioral Equations

(1) $Y_t^w = P_t^c C_t / W_t + P_t^i I_t / W_t$
$= C_t^w + I_t^w,$

(2) $M_t^{wd} = m^d(Y_t^w, R_t), \quad m_1^d > 0, \; m_2^d < 0,$

(3) $M_t^{ws} = m^s(R_t), \qquad m^{s'} > 0,$

(4) $A_t^{ws} = P_t^a A_t / W_t$
$= A_t^w,$

(5) $A_t^{wd} = a^d(P_t^a, R_t, Y_t^w), \qquad a_1^d, \; a_2^d < 0, \quad a_3^d > 0.$

(6) $A_t^w = a^d(P_t^a, R_t, Y_t^w),$

(7) $P_t^a = a(R_t, A_t^w, Y_t^w), \quad a_1, a_2 < 0, \; a_3 > 0.$

(8) $P_t^{cs} = c^{sp}(C_t^w), \qquad c^{sp'} > 0.$

(9) $P_t^{cd} = c^{dp}(C_t^w, Y_t^w), \; c_1^{dp} < 0, \; c_2^{dp} > 0.$

(10) $c^{sp}(C_t^w) = c^{dp}(C_t^w, Y_t^w) = P_t^c,$

(11) $C_t^{wd} = c(Y_t^w), \quad 0 < c' < 1,$

(12) $S_t^w = Y_t^w - c(Y_t^w)$
$= s(Y_t^w), \quad 0 < s' < 1,$

(13) $P_t^{is} = i^{sp}(I_t^w), \qquad i^{sp'} > 0.$

(14) $P_t^{id} = i^{dp}\left(I_t^w, R_t, P_t^a, c(Y_t^w)\right)$
$= i^{dp}(I_t^w, R_t, P_t^a, Y_t^w), \quad i_1^{dp}, \; i_2^{dp} < 0, \; i_3^{dp} \; i_4^{dp} > 0.$

(15) $i^{sp}(I_t^w) = i^{dp}(I_t^w, R_t, P_t^a, Y_t^w) = P_t^i,$

(16) $I_t^{wd} = i(R_t, P_t^a, Y_t^w), \qquad i_1 < 0, \; i_2, i_3 > 0,$

(17) $Y_t^{wd} = C_t^{wd} + I_t^{wd}$
$= c(Y_t^w) + i(R_t, P_t^a, Y_t^w),$

(18) $N_t^{wid} = n^{id}(I_t^{we}), \qquad n^{id'} > 0,$

(19) $N_t^{wcd} = n^{cd}(C_t^{we}), \qquad n^{cd'} > 0,$

$$(20)\quad N_t^w = n^{id}(Y_t^{we}) + n^{cd}(Y_t^{we}) = n(Y_t^{we}), \qquad n' = 1,$$

$$(21)\quad Y_t^{we} = C_t^{we} + I_t^{we}.$$

$$(22)\quad Y_t^{ws} = n^{-1}(N_t^w), \quad n^{-1'} = 1,$$

$$(23)\quad Y_t^{wd} = c\left(n^{-1}(N_t^w)\right) + i\left(R_t, P_t^a, n^{-1}(N_t^w)\right) = y^d(N_t^w, R_t, P_t^a), \quad y_1^d, y_3^d > 0,\ y_2^d < 0,$$

I I. Dynamic Adjustment Functions

$$(24)\quad dR_t = g^r\left(M_t^{wd} - M_t^{ws}\right) = g^r\left(m^d(Y_t^w, R_t) - m^s(R_t)\right)$$

$$(25)\quad dM_t = g^r\left(M_t^{wd} - M_t^w\right) = g^r\left(m^d(Y_t^w, R_t) - M_t^w\right)$$

$$(26)\quad dP_t^a = g^{pa}\left(A_t^{wd} - A_t^{sw}\right) = g^{pa}\left(a^d(P_t^a, R_t, Y_t^w) - A_t^w\right).$$

$$(27)\quad dC_t^{we} = g^{ce}\left(C_t^{wd} - C_t^{we}\right) = g^{ce}\left(c^{dp-1}(P_t^c, Y_t^w) - C_t^{we}\right)$$

$$(28)\quad dI_t^{we} = g^{ie}\left(I_t^{wd} - I_t^{we}\right) = g^{ie}\left(i^{dp-1}\left(P_t^i, R_t, P_t^a, Y_t^w\right) - I_t^{we}\right)$$

$$(29)\quad dC_t^w = g^c(C_t^{we} - C_t^w)$$

$$(30)\quad dI_t^w = g^i(I_t^{we} - I_t^w)$$

$$(31)\quad dP_t^i = g^{pi}\left(I_t^d - I_t^s\right) = g^{pi}\left(i^{dp-1}\left(P_t^i, R_t, P_t^a, Y_t^w\right) - i^{sp-1}\left(P_t^i\right)\right),$$

$$(32)\quad dP_t^c = g^{pc}\left(C_t^d - C_t^s\right) = g^{pc}\left(c^{dp-1}(P_t^c, Y_t^w) - c^{sp-1}(P_t^c)\right).$$

$$(33)\quad dN_t^w = g^{nd}(Y_t^{we} - Y_t^{ws}) = g^{nd}\left(Y_t^{we} - n^{-1}(N_t^w)\right)$$

$$(34)\quad dY_t^{we} = dC_t^{we} + dI_t^{we}$$
$$= g^{ce}\left(c^{dp-1}\left(P_t^{cd}, Y_t^w\right) - C_t^{we}\right) + g^{ie}\left(i^{dp-1}\left(P_t^{id}, R_t, P_t^a, Y_t^w\right) - I_t^{we}\right)$$
$$(35)\quad dY_t^w = dC_t^w + dI_t^w$$
$$= g^c(C_t^{we} - C_t^w) + g^i(I_t^{we} - I_t^w).$$

III. Structure of Keynes' Aggregate Model

Table 1: Structure of Keynes' Aggregate Model			
Market	**Equilibrium Conditions**		**Endogenous Variables**
Assets	$M_t^{ws} = M_t^w$	$M_t^{ws} = M_t^{wd}$	$M_t^w,\ R_t,\ P_t^a$
	$A_t^{wd} = A_t^w$		
Investment	$I_t^{we} = I_t^{wd}$	$I_t^{we} = I_t^w$	$I_t^{we},\ I_t^w, P_t^i$
	$P_t^{id} = P_t^{is}$		
Consumption	$C_t^{we} = C_t^{wd}$	$C_t^{we} = C_t^w$	$C_t^{we},\ C_t^w,\ P_t^c$
	$P_t^{cd} = P_t^{cs}$		
Labor	$Y_t^{ws} = Y_t^{we}$		N_t^w
Identities	$Y_t^{we} = C_t^{we} + I_t^{we}$		Y_t^{we}
	$Y_t^w = C_t^w + I_t^w$		Y_t^w

Figure 1: Short-Run Equilibrium.

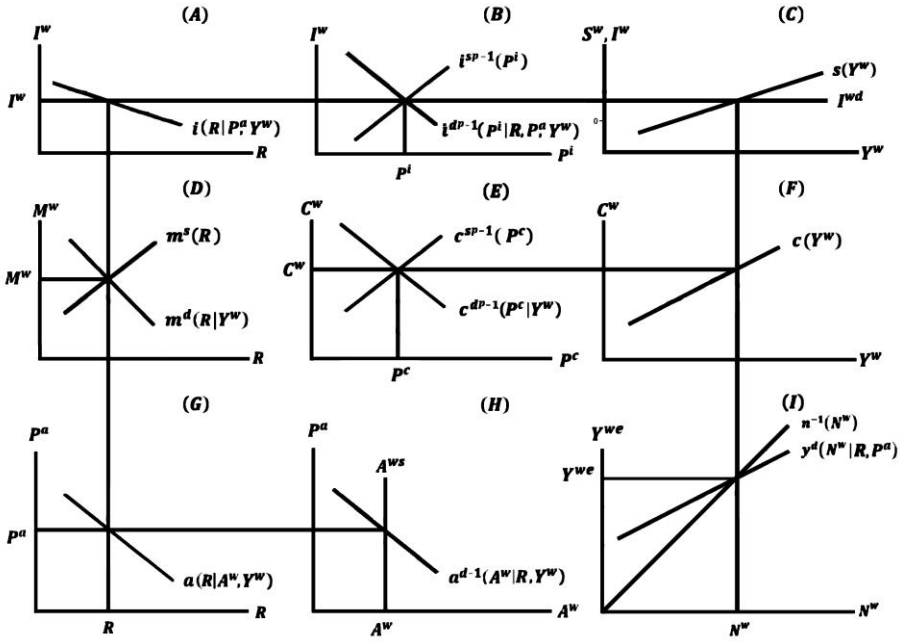
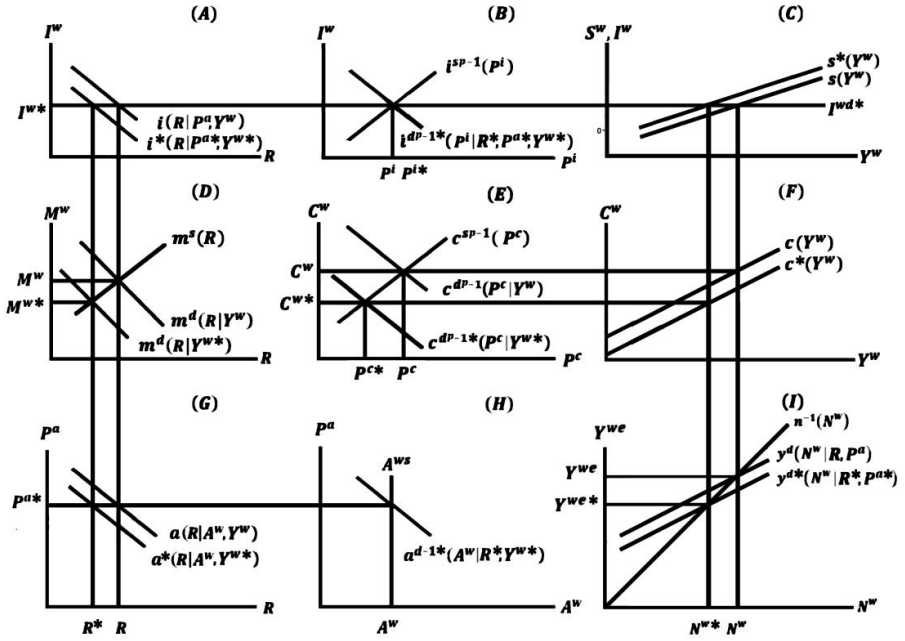


Figure 2: Achieving Short-Run Equilibrium.



V. Long-Run Equilibrium

$$(14a)\quad P_t^{id} = i^{dp}(I_t^w, R_t, A_t^w, P_t^a, Y_t^w), \qquad i_3^{dp},\ i_5^{dp} > 0,\ \ i_1^{dp},\ i_2^{dp},\ i_3^{dp} < 0.$$

$$(16a)\quad I_t^{wd} = i(R_t, A_t^w, P_t^a, Y^w), \qquad i_3^d, i_4^d > 0,\ \ i_1^d,\ i_2^d < 0$$

$$(23a)\quad Y_t^{wd} = c\left(n^{-1}(N_t^w)\right) + i\left(R_t, A_t^w, P_t^a, n^{-1}(N_t^w)\right)$$

$$= y^d(N_t^w, R_t, A_t^w, P_t^a), \qquad y_1^d, y_4^d > 0,\ y_2^d, y_3^d < 0.$$

$$(8a)\quad P_t^{cs} = P^{c*}$$

$$(13a)\quad P_t^{is} = P^{i*}.$$

$$(28a)\quad dI_t^{we} = g^{ie}\left(I_t^{wd} - I_t^{we}\right)$$

$$= g^{ie}\left(i^{dp-1}\left(P_t^i, R_t, A_t^w, P_t^a, Y_t^w\right) - I_t^{we}\right)$$

$$(34a)\quad dY_t^{we} = dC_t^{we} + dI_t^{we}$$

$$= g^{ce}\left(c^{dp-1}(P_t^c, Y_t^w) - C_t^{we}\right) + g^{ie}\left(i^{dp-1}\left(P_t^i, R_t, A_t^w, P_t^a, Y_t^w\right) - I_t^{we}\right).$$

Figure 3: Toward Long-Run Equilibrium.